International Journal of Theoretical Physics, Vol. 31, No. 2, 1992

Inhomogeneous Static Model in Brans–Dicke Theory

Marcelo Samuel Berman[1]

Received May 1, 1991

A static universe with position-dependent rest-energy density, pressure, and scalar field is considered in Brans–Dicke theory. A perfect-gas equation of state is obtained with the solution to the field equations for the Euclidean case with Robertson–Walker metric.

Brans–Dicke (1961) theory is a viable alternative to general relativity when the coupling constant $\omega > 500$. The field equations of this theory are

$$G_j^i = 8\pi\phi^{-1}T_j^i + \omega\phi^{-2}(\phi^i\phi_j - \tfrac{1}{2}\delta_j^i\phi^k\phi_k) + \phi^{-1}(\phi^i_{;j} - \delta_j^i\phi^k_{;k}) \tag{1}$$

and

$$(3+2\omega)\phi^k_{;k} = 8\pi T \tag{2}$$

where G_j^i is the Einstein tensor, T_j^i is the stress-energy tensor, and

$$T \equiv T_k^k \tag{3}$$

$$\phi_i \equiv \frac{\partial\phi}{\partial x^i} \tag{4}$$

For a Robertson–Walker metric for the spatially flat case,

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2\,d\theta^2 + r^2\sin^2\theta\,d\psi^2) \tag{5}$$

the field equations reduce, in the static case, when

$$\dot{a}(t) \equiv 0 \tag{6}$$

[1]Department of Physics and Astronomy, University of Alabama, Tuscaloosa, Alabama 35487-0324.

0020-7748/92/0200-0321$06.50/0

to the following relations:

$$p\frac{a^2}{\phi}=\frac{2}{r}\frac{\phi'}{\phi}-\frac{\omega}{2}\frac{(\phi')^2}{\phi^2}=\frac{\phi''}{\phi}+\frac{\phi'}{r\phi}+\frac{\omega}{2}\frac{(\phi')^2}{\phi^2} \tag{7}$$

$$\rho\frac{a^2}{\phi}=-\frac{\phi''}{\phi}-\frac{2}{r}\frac{\phi'}{\phi}-\frac{\omega}{2}\frac{(\phi')^2}{\phi^2} \tag{8}$$

$$(3+2\omega)\left(\frac{\phi''}{\phi}+\frac{2\phi'}{r\phi}\right)=(3p-\rho)\frac{a^2}{\phi} \tag{9}$$

where p and ρ stand for cosmic pressure and rest-energy density, respectively, and T^i_j is for a perfect fluid case.

If we suppose, as a tentative solution,

$$\phi=Ar^n \qquad (A, n \text{ constants}) \tag{10}$$

we find the following solution:

$$\omega=2/n-1 \tag{11}$$

$$n=3\pm\sqrt{13} \tag{12}$$

$$\rho=\frac{n}{a^2r^2}\left(4-\frac{n}{2}\right)\phi \tag{13}$$

Though neither ρ nor p is a constant, they obey a perfect-gas-law equation of state, i.e.,

$$p=\alpha\rho \qquad (\alpha=\text{const}) \tag{14}$$

This position-dependent solution for ϕ, ρ, and p should be compared with the results obtained in the homogeneous case by Berman *et al.* (1989). For that case, a flat solution required a time-varying ϕ function, given by

$$\phi=-\frac{4\pi\rho_0}{\omega}(t+C)^2 \qquad (C=\text{const}) \tag{15}$$

where

$$\rho=\rho_0=\text{const} \tag{16}$$

$$p=p_0=\text{const} \tag{17}$$

Contact with Newton's gravitational constant G can be made by noting that

$$G=\frac{2\omega+4}{2\omega+3}\phi^{-1} \tag{18}$$

so that we have, in our case,

$$G = \frac{2\omega + 4}{2\omega + 3} A^{-1} r^{-n} \tag{19}$$

Berman *et al.* (1989) commented that the study of static universes is appealing, not only for theoretical reasons, but also because, if there exists an explanation for the observed red-shift other than the expansion of the universe, such study would gain importance. Peratt (1990) has offered insight into such a possible scenario based on plasma physics studies.

ACKNOWLEDGMENTS

I am grateful to Prof. M. M. Som for discussing this and other articles. This work was partially funded by CNPq (Brazil).

REFERENCES

Berman, M. S., Som, M. M., and Gomide, F. M. (1989). *General Relativity and Gravitation*, **21**, 287.

Brans, C., and Dicke, R. H. (1961). *Physical Review*, **124**, 925.

Peratt, A. L. (1990). *The Sciences* (N.Y. Acad. Sci.), No. 1, 24.